THOUGHT EXPERIMENT

Given: a universe where it is proven that **P** ≠ **NP** (Assumption 1)
Given: a graph G
Consider: a maximum matching M, a minimum vertex cover S

Fact 1: |M| ≤ |S|
Fact 2: Finding M is in **P**
Fact 3: Finding S is **NP-complete**

World 1
- We are not given S, M.
- We do not know anything about finding S, hence we are trying to understand it better. We start by understanding its bound.

(This simulates how research on solving most open problems proceeds.)

Problem 1: Find a lower bound on the size of S for a graph G.
Solution 1: Find a maximum matching M. |M| is a lower bound [Fact 1].

Conclusion 1.1: Finding a bound on the size of an **NP-complete** problem is in **P**.
Conclusion 1.2: Finding a bound on the size of a problem is easier than finding the exact value.

(Both conclusions seem logical.)

World 2
- We are not given M, S.
- We do not know anything about finding M, hence we are trying to understand it better. We start by understanding its bound.

(This simulates how research on solving most open problems proceeds.)

Problem 2: Find an upper bound on the size of M for a graph G.
Solution 2: Find a min vertex cover S. |S| is an upper bound [Fact 1].

Conclusion 2.1: Finding a bound on the size of a problem in **P** is **NP-complete**.
Conclusion 2.2: Finding a bound on the size of a problem is harder than finding the exact value.

(Both conclusions defy logic! Shouldn't finding a bound be at least as easy as finding the exact number, in line with most math research?)

Disclaimer: for the purposes of this thought experiment, we interchangeably use the decision version and the search version of a problem, and NP-hardness and NP-completeness.

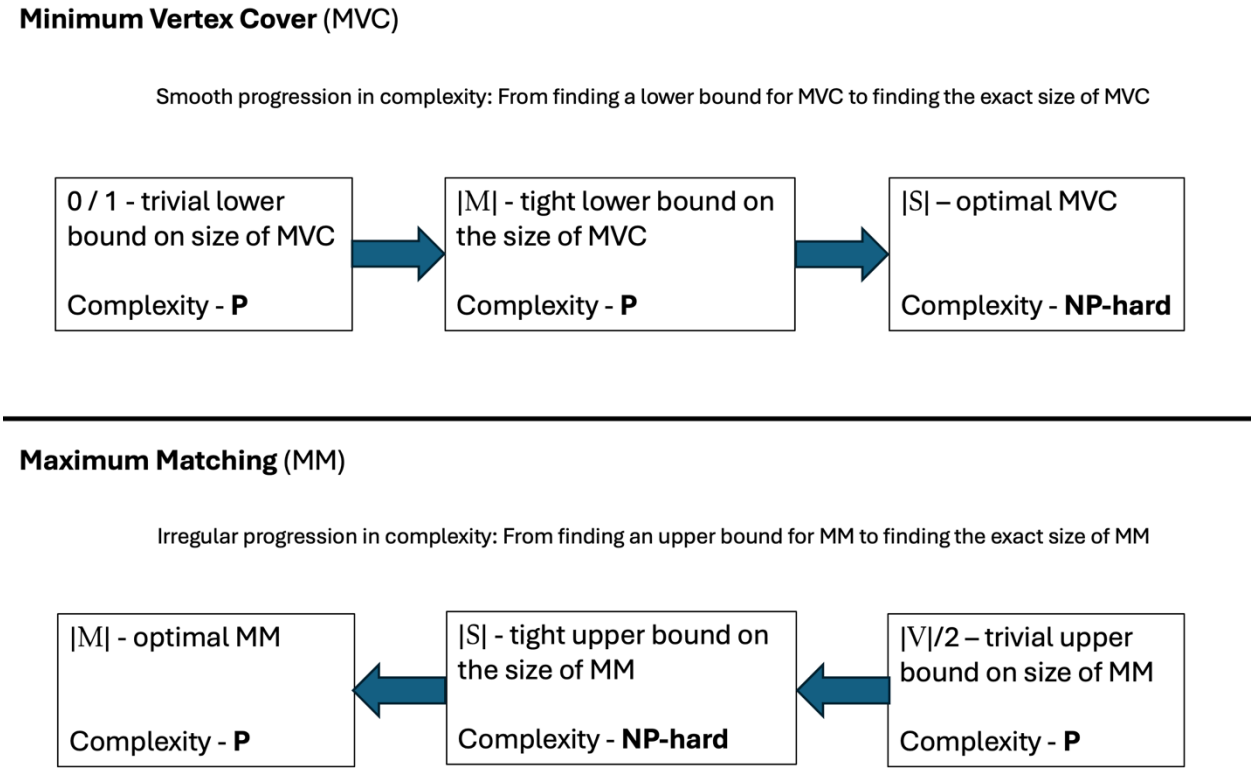

Figure 1: An illustration depicting progression of computational complexity from finding trivial bound to exact value for (top) the minimum vertex cover problem and (bottom) the maximum matching problem. The contradictory / anomalous nature of the progression of complexity in the bottom problem and the corresponding conclusions (2.1 and 2.2) seems to defy logic when compared to the top problem and the corresponding conclusions (1.1 and 1.2). This is especially true given that one can find a matching M to make Problem 2 efficient. If P=NP, such anomalies do not exist.